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IMPROVED DATA AGREEMENT USING NEW EDDY VISCOSITY EQUATIONS IN A COAXIAL FREE-JET COMPUTER CODE

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16. Abstract
An existing boundary layer flow code was modified to improve the prediction of concentration and velocity profiles in two-fluid coaxially flowing free jets. Calculations were compared with air data for outer-stream to jet-stream velocity ratios of 3.4 to 39.5. These velocity ratios are expected in a coaxial flow gas core nuclear rocket engine. The inlet velocity profile strongly influenced the calculations. The best data fit was obtained with a continuous, two-region, axial formulation of eddy viscosity with no radial dependence. The evaluated coefficients were found to be independent of velocity ratio.

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SUMMARY

The main fluid mechanics problem in a coaxial flow gas core nuclear rocket is that of minimizing the nuclear fuel transport, by turbulent mixing, from the central fuel jet to the higher velocity coaxial propellant stream while maximizing the outer-stream to jet-stream velocity ratio. These requirements oppose each other and also strongly affect rocket performance, so that some proper balance is important to achieve. Therefore, an accurate means of predicting fuel concentration and velocity profiles is needed for system optimization studies.

An existing coaxial flow free-jet computer code is available for this purpose. Its accuracy is limited by the fact that the eddy viscosity is not accurately known. In the present study this jet code is modified and the eddy viscosity formulation that gives best agreement with pure air data is determined. The outer-stream to jet-stream velocity ratios investigated ranged from 3.4 to 39.5. Preliminary calculations indicated that the specified inlet velocity profile was important. The measured profile was used as code input for the data comparison.

From calculations with various eddy viscosity expressions in the code it was concluded that a continuous two-region eddy viscosity formulation gave closest data agreement. A radial variation of eddy viscosity was found to be unimportant. The two-region formulation consisted of a half-jet expression near the inlet and a velocity defect expression downstream. Two coefficients in these expressions were evaluated to give the best data fit, and they were found to be independent of velocity ratio.

The largest remaining discrepancy between calculations and data occurred at the highest velocity ratio (39.5) near the inlet (within 5.6 jet radii) where the jet width was underestimated by up to 40 percent. This discrepancy is attributed to the limitation of the boundary layer solution code, which does not account for recirculation at the inlet as was observed experimentally at velocity ratios of 16.0 and larger.

These results also apply in general to the coaxial free jet with different density fluids, for which the two coefficients in the eddy viscosity expressions may have to be reevaluated for more precise data representation.

INTRODUCTION

A quantitative understanding of coaxial turbulent jet mixing is important for optimizing performance in many applications ranging from combustion chambers and jet pumps to the gas core nuclear rocket (ref. 1). One jet configuration that has recently been studied in detail (refs. 2 to 5) is that of a free jet with a faster moving coaxial outer stream (see fig. 1). This jet configuration, which is particularly important in the coaxial flow gas core nuclear rocket concept, requires further study for accurately predicting jet mixing effects on rocket performance.

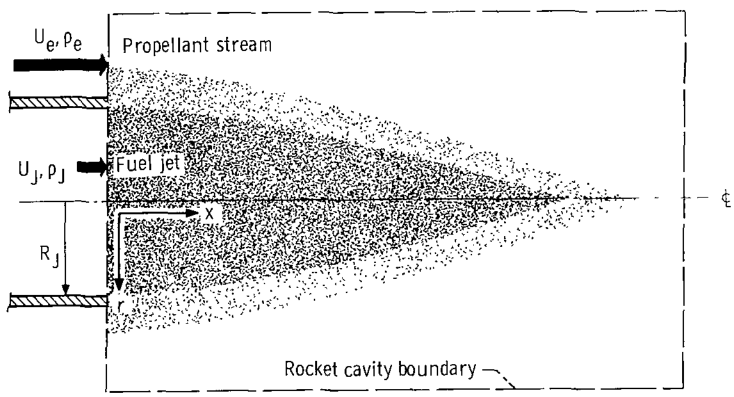


Figure 1. - Model of coaxial free jet as applied to coaxial flow gas core nuclear rocket.

In one concept of a gas core nuclear rocket a high velocity hydrogen propellant stream surrounds a uranium vapor fuel jet (see fig. 1). The propellant stream is heated to about 10 000° R by thermal radiation from the fissioning fuel core at about 100 000° R, and it delivers a specific impulse of 1500 to 2000 seconds. The fuel-to-propellant density ratio ranges from 1.0 to 4.0. The outer-stream to jet-stream velocity ratio ranges from 10 to 100. The coaxial jet provides nuclear fuel containment at these extremely high temperatures without the usual fuel element temperature limitations. In the proposed rocket the jet is enclosed in a cylindrical cavity at a pressure of 1000 atmospheres. This cavity has a diameter of about 4 jet radii and a length of up to 10 jet radii.

The main fluid mechanics problem in the gas core nuclear rocket is that of minimizing the turbulent transport of fuel from the jet stream to the outer stream while maximizing the outer-stream to jet-stream velocity ratio (see ref. 6). These requirements oppose each other and strongly affect rocket performance. Therefore, an accurate means of predicting fuel concentration and velocity profiles is needed for parametric and system optimization studies.

In this report the influence of the cavity walls is not considered. Free-jet flow is assumed. In addition, it is assumed that the flow is isothermal.

A similarity solution for the turbulent single-fluid coaxial jet with a faster moving free stream has been published by Donovan (ref. 3). This solution, which gives good agreement with pure air data in the downstream region ($x/R_J > 10$), does not apply near the inlet. In order to account for the nonsimilar flow near the inlet, a computer solution of the two-fluid boundary layer equations has been written by Donovan and Todd (ref. 4). For the eddy viscosity assumptions and step inlet velocity profiles in this computer code, comparison with the recent single-fluid and two-fluid data of Zawacki and Weinstein (ref. 5) indicated that this code significantly overestimated the axial decay rates. The code therefore underestimated the jet width, as described by the half radius (the value of r where $(U_e - u)/(U_e - U_J) = 1/2$). Experience with this and other turbulent jet flow codes indicated that the code accuracy would be improved by properly adjusting the empirical eddy viscosity in the code.

The main objective of this report is to find eddy viscosity expressions, for use in the jet code of reference 4, so that the calculations will agree more closely with measured velocity and concentration profile data near the jet inlet ($x/R_J < 10$). The importance of the inlet velocity profile in the code is also studied. Comparison is made mostly with single fluid data from reference 5.

Specific questions to be answered by this study are the following:

(1) How important is the jet inlet velocity profile, specified in the computer code, for a valid comparison with experimental data?

(2) What is the best far-jet eddy viscosity axial formulation?

(3) What is the best near-jet eddy viscosity axial formulation?

(4) How should the near-jet and far-jet eddy viscosity formulations be combined (i.e., where is the near- to far-jet cutoff)?

(5) Must the radial variation of eddy viscosity be accounted for?

(6) For what range of inlet velocity ratios and distances from the inlet does the final modified code give good agreement with data?

SYMBOLS

K	coefficient for constant eddy viscosity
k_0	coefficient for upstream turbulence in near-jet eddy viscosity
k_1	coefficient in near-jet eddy viscosity
k_2	coefficient in far-jet eddy viscosity
R_J	jet inlet radius
r	radial distance from centerline

$r_{1/2}$ value of r where $(U_e - u)/(U_e - U_J) = 0.5$

Sc_t turbulent Schmidt number, ratio of eddy mass diffusivity to eddy viscosity

$U_{\mathfrak{t}}$ axial velocity at centerline

U_e outer stream velocity

U_J average jet velocity

u axial velocity

v radial velocity

x axial distance from inlet

x_{12} end of near-jet region, start of far-jet region

y mass fraction of jet fluid

δ thickness of outer stream boundary layer on the jet tube

ϵ axially dependent eddy viscosity

ϵ_r radially dependent eddy viscosity

ρ fluid density

ρ_0 reference fluid density

ANALYSIS

Boundary Layer Equation Solution

A computer code has been written by Donovan and Todd (ref. 4) for calculating the two-fluid coaxial jet mixing with a moving free stream. For the convenience of the reader, the important features of the code that apply to this report are given here. This code allows for the higher velocity free stream and the higher density jet that are typical of the gas core nuclear rocket. For the fluid mechanics model of the coaxial flow rocket concept, see figure 1. The solution makes use of the boundary layer assumptions. The programmed equations are as follows:

Continuity equation:

$$\frac{\partial}{\partial x}(\rho u r) + \frac{\partial}{\partial r}(\rho v r) = 0 \tag{1}$$

x-Momentum equation:

$$\rho u \frac{\partial u}{\partial x} + \rho v \frac{\partial u}{\partial r} = \frac{1}{r} \frac{\partial}{\partial r} \left(\rho \epsilon r \frac{\partial u}{\partial r} \right) \tag{2}$$

Mass diffusion equation:

$$\rho u \frac{\partial y}{\partial x} + \rho v \frac{\partial y}{\partial r} = \frac{1}{r} \frac{\partial}{\partial r} \left(\frac{\rho \epsilon}{Sc_t} r \frac{\partial y}{\partial r} \right) \tag{3}$$

The initial and boundary conditions are

$$\left. \begin{array}{l} 0 \leq r < R_J; \ u = U_J, \ y = 1.0 \\ r > R_J; \ u = U_e, \ y = 0 \end{array} \right\} \text{for } x = 0$$

and

$$\left. \begin{array}{l} r = 0; \ \dfrac{\partial u}{\partial r} = 0, \ v = 0, \ \dfrac{\partial y}{\partial r} = 0 \\ r \rightarrow \infty; \ u \rightarrow U_e, \ y \rightarrow 0 \end{array} \right\} \text{for } x \geq 0$$

The original eddy viscosity formulations used in this code were

$$\epsilon = \epsilon_1 = k_1 x (U_e - U_J) \left[\frac{\rho_0}{\rho} \right]^2 \qquad \text{for } \frac{U_e - U_{\mathfrak{t}}}{U_e - U_J} \geq 0.99 \tag{4}$$

and

$$\epsilon = \epsilon_2 = k_2 r_{1/2} (U_e - U_{\mathfrak{t}}) \left[\left(\frac{\rho_0}{\rho} \right)^2 \frac{1}{r^2} \int_0^r 2 \frac{\rho}{\rho_0} r \, dr \right] \qquad \text{for } \frac{U_e - U_{\mathfrak{t}}}{U_e - U_J} < 0.99 \tag{5}$$

where ρ_0 was to be determined experimentally. No value for ρ_0 was given in reference 4.

Reference 4 gives the values of $k_1 = 0.00075$ and $k_2 = 0.0256$. Also given is the near-jet cutoff x_{12} at the dimensionless centerline velocity of 0.99. These values were determined by the best match of an ambient-jet similarity solution with the computer solution in the region $x/R_J > 10$. The bracketed quantities in equations (4) and (5) are adjustments for variable density flows made by using the Ting-Libby formulation (ref. 7). These adjustments were not verified in reference 4.

Modifications to Computer Code

The modifications to the computer code that are studied in this report are
(1) Jet and outer stream inlet velocity profile
(2) Axial formulation of eddy viscosity
(3) Radial formulation of eddy viscosity

Inlet Velocity Profile

The inlet velocity profiles studied are
(1) The step profile with uniform inlet velocity in the jet, and a uniform, but larger, inlet velocity in the outer stream (see fig. 2)

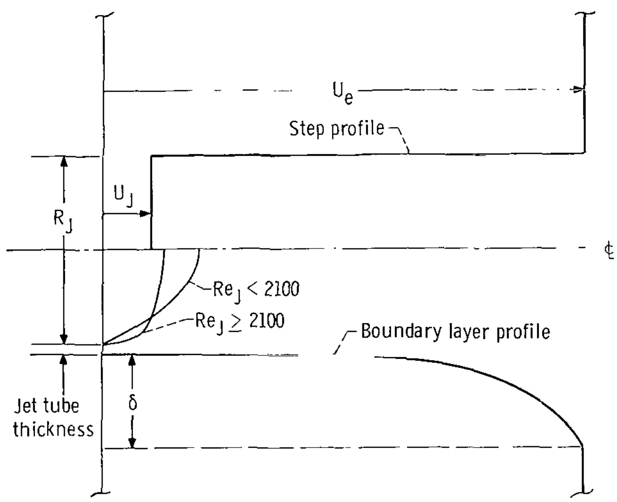


Figure 2. - Inlet velocity profiles for computer code.

(2) The boundary layer inlet velocity profile (see fig. 2) with a one-seventh power velocity profile in the outer stream boundary layer on the jet tube, and with a parabolic or one-seventh power velocity profile in the jet, depending on the jet pipe Reynolds number being less or greater than 2100 (see fig. 2)
(3) The boundary layer inlet velocity profile with a finite thickness jet tube

Eddy Viscosity Axial Formulation

Two types of axial formulations are studied. The first is a simple one-region expression

$$\epsilon = KR_J U_J = \text{Constant} \qquad \text{for all } x \tag{6a}$$

where the value of K has been correlated for an air-bromine jet in reference 2 as

$$K = 0.0344\left(\frac{U_e}{U_J} - 1\right)^{1/2}\left(1 - \frac{250}{Re_J}\right) \tag{6b}$$

The second axial formulation is a two-region formulation which recognizes the different flow structure near the inlet and far downstream. The near-jet expression is similar to Goertler's half-jet expression (see Schlichting, ref. 8, p. 598) with an extra term for turbulence upstream of the inlet:

$$\epsilon_1 = k_0 R_J U_J + k_1 x(U_e - U_J) \qquad \text{for } x \leq x_{12} \tag{7a}$$

The far-jet expression is the familiar Prandtl velocity defect expression (ref. 8, p. 592)

$$\epsilon_2 = k_2 r_{1/2}(U_e - u_{\mathrm{\not{c}}}) \qquad \text{for } x > x_{12} \tag{7b}$$

The near-jet cutoff x_{12} is determined by either of these cutoff conditions: (1) the conventional potential core end condition as used in reference 4 with

$$x_{12} \text{ being the location where } \frac{U_e - U_{\mathrm{\not{c}}}}{U_e - U_J} = 0.99 \tag{7c}$$

or (2) a continuous eddy viscosity condition with

$$x_{12} \text{ being the location where } \epsilon_1 = \epsilon_2 \tag{7d}$$

The coefficients k_0, k_1, and k_2 in equations (7a) and (7b) are evaluated in the next section by a comparison with air-air data from Zawacki and Weinstein (ref. 5).

Eddy Viscosity Radial Variation

The importance of radial variation of eddy viscosity is studied for the simple cosine expression

$$\epsilon_r = \frac{2\epsilon}{3}\left(1 + \cos\frac{\pi}{3}\frac{r}{r_{1/2}}\right) \tag{8}$$

RESULTS AND DISCUSSION

A well-known fact of jet theory is that most jet velocity profiles, when properly normalized, can be plotted on the same gaussian curve (see ref. 9, ch. 1). In the case of the coaxial free jet, this universal relation has the form

$$\frac{U_e - u}{U_e - U_{\mathbf{\mathfrak{t}}}} = f\left(r/r_{1/2}\right)$$

where $r_{1/2}$ is defined to be the value of r corresponding to

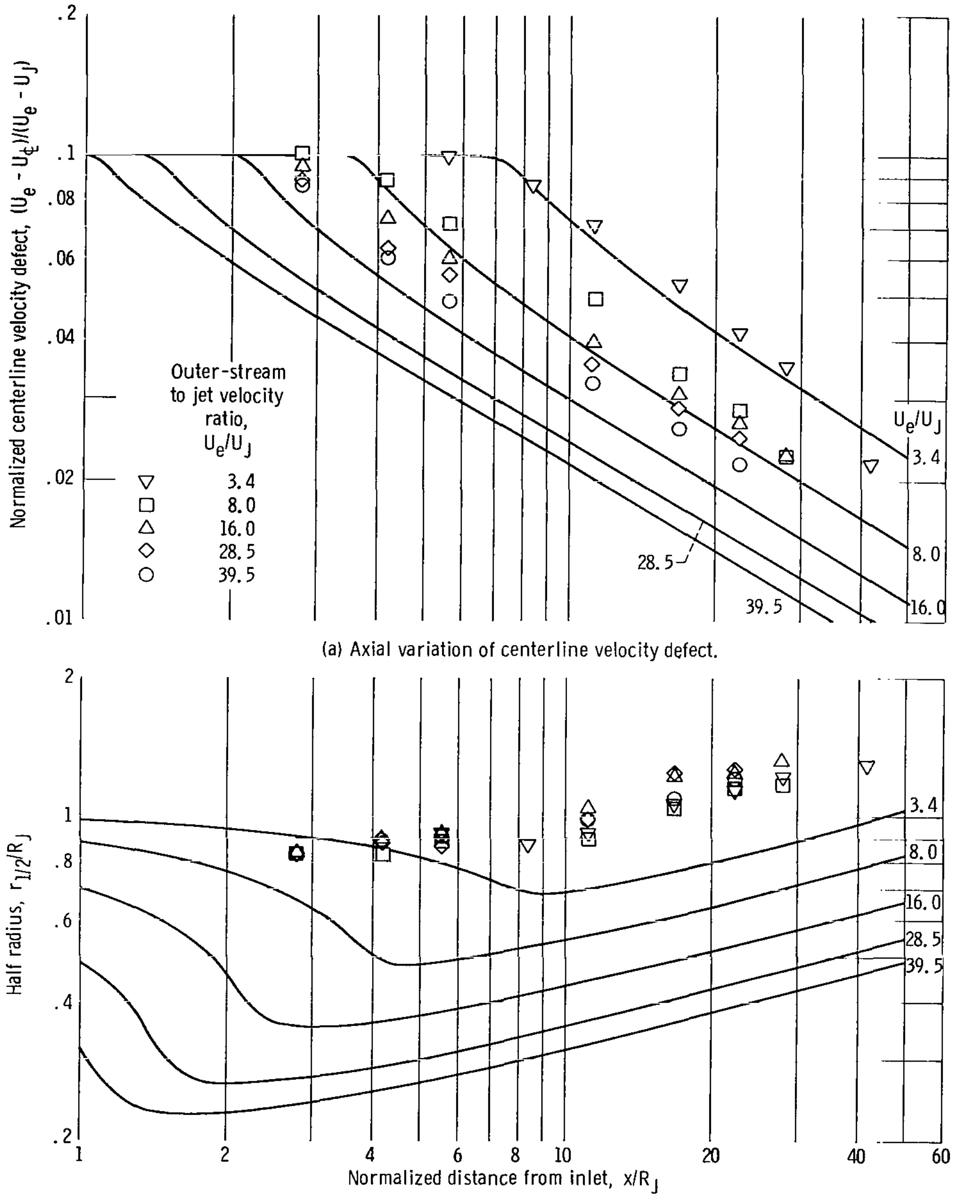
$$(U_e - u) = 1/2(U_e - U_{\mathbf{\mathfrak{t}}})$$

When this fact is used, the velocity field in the coaxial free jet is most simply and completely described by plotting the axial variations of the dimensionless quantities $(U_e - U_{\mathbf{\mathfrak{t}}})/(U_e - U_J)$ and $r_{1/2}$.

In order to show the extent of discrepancy between the original computer code of reference 4 and the experimental data of reference 5, the calculated and measured values of centerline velocity defect and half radius are plotted in figures 3(a) and (b) for the inlet velocity ratios of $U_e/U_J = 3.4$, 8.0, 16.0, 28.5, and 39.5. Figures 3(a) and (b) indicate that the value of x for a fixed centerline velocity defect is underestimated by as much as a factor of 2.3, and the calculated half radius is too small by as much as a factor of 3.5. These discrepancies are greatest for the largest velocity ratio $U_e/U_J = 39.5$.

Several features of the experimental data should be noted. For the velocity ratios $U_e/U_J = 16$, 28.5, and 39.5, Zawacki and Weinstein (ref. 5) observed large velocity fluctuations at the inlet plane of the jet tube. These fluctuations had dissipated at $x/R_J = 5.6$. These investigators concluded that the fluctuations were caused by the sudden inflow from the high velocity outer stream into the jet, and the formation of a recirculating region at the jet inlet. Hot-wire anemometer velocity measurements were made in this region, but no pressure measurements were made. The recirculation phenomena was not further explored in reference 5. More recent unpublished work on this same flow rig has further confirmed the presence of the inlet recirculation at velocity ratios $U_e/U_J \geq 16$ and has also found the velocity values of reference 5 near the axis at $x/R_J = 1.4$ to be in error. These velocity values are therefore not used for data comparison in this report.

Another important fact is that the experimental data were taken in an 8 inch square duct with a 3/4 inch diameter jet tube, whereas the computer code calculates unducted flow. An estimate of the boundary layer on this outer stream duct indicates that within



(a) Axial variation of centerline velocity defect.

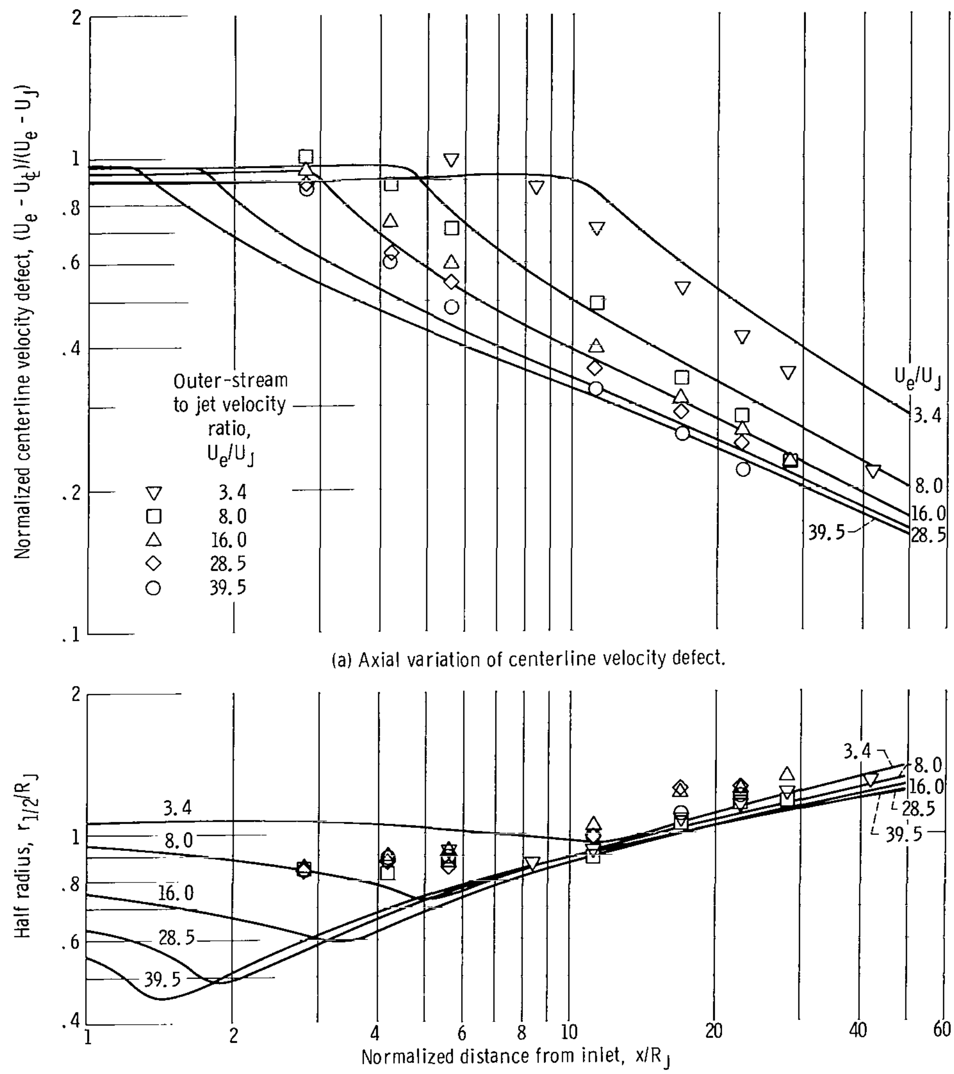
(b) Axial variation of half radius.

Figure 3. - Centerline velocity defect and half-radius variation for various inlet velocity ratios. Code from reference 4 unmodified; data from reference 5.

the range of this data, $x/R_J < 50$, this boundary layer thickness is only a small fraction of the duct cross section. Thus the data are assumed to apply for free-jet flow.

Inlet Velocity Profile

In order to study the importance of inlet velocity profile in the calculations, the computer code of reference 4 was modified to permit shaping of the inlet velocity profiles. This modification involved reprogramming a subroutine that calculates stream-function as



(b) Axial variation of half radius.

Figure 4. - Velocity defect and half-radius variation for various inlet velocity ratios. Code from reference 4 with boundary layer inlet velocity profile; data from reference 5.

a function of radius at the jet inlet for a specified inlet velocity profile. In addition to the original step profile, calculations were run for the measured boundary layer inlet profile shown in figure 2 and described in the ANALYSIS section. For this profile the boundary layer thickness on the outside of the jet tube, which was a strong feature of the data, was estimated as $\delta = R_J$. This value does not change with velocity ratio U_e/U_J in the data since U_e was fixed and only U_J was varied. The jet inlet profile was taken as laminar for $U_e/U_J > 8.0$, where $Re_J < 2100$. The specified inlet velocity profile also accounted for a finite jet pipe thickness. However, the jet pipe thickness of reference 5, which was only $0.04\ R_J$, was found to give the same results as a zero pipe thickness.

The sensitivity of the calculations to the specified inlet velocity profile is best illustrated by comparing calculations for both the step and boundary layer inlet velocity profiles with the experimental data. The results for the step profile are shown in figure 3, and those for the boundary layer profile are shown in figure 4. For this comparison, the same eddy viscosity (from ref. 4) is used in figures 4 and 5. The agreement in centerline velocity defect has been somewhat improved in figure 4(a) as compared to figure 3(a). The agreement in half radius with the boundary layer profile as shown in figure 4(b) is, however, greatly improved as compared to figure 3(b). The half radius, which was underestimated by as much as a factor of 3.5 in figure 3(b), is now at most a factor of 1.5 too small. The effect of the specified inlet velocity profile is seen to be strongest for downstream half radius at the large inlet velocity ratios.

Thus it is concluded that the inlet velocity profile strongly affects the calculations and must be accounted for in the computer code. The inlet velocity profile is especially

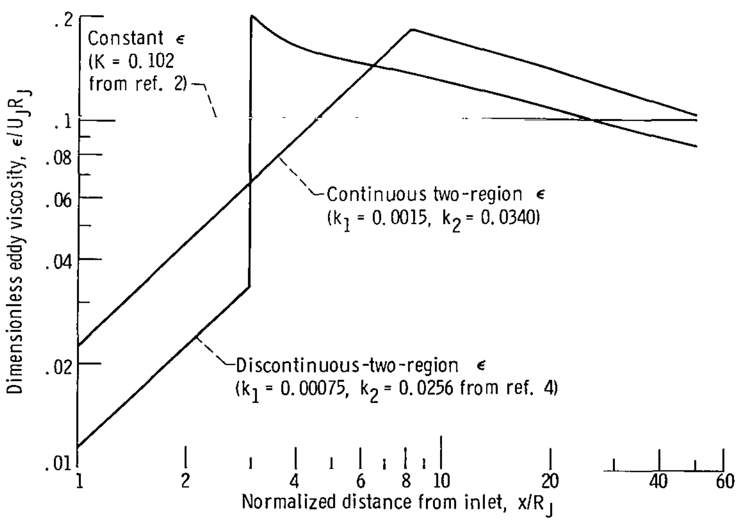


Figure 5. - Various axial formulations of eddy viscosity. Inlet velocity ratio, $U_C/U_t = 16.0$.

important at the large velocity ratios. For the remainder of this study only the boundary layer inlet profile is considered.

Eddy Viscosity Axial Formulation

Most expressions for jet eddy viscosity found in the literature are based on jet similarity arguments which do not simultaneously apply for flow near the inlet and downstream. These eddy viscosity expressions generally take the form of a simple constant value (eq. (6)) or the more sophisticated Prandtl velocity defect expression (eq. (7b)). These similarity expressions give sufficient accuracy if only the downstream flow (beyond about $x/R_J = 10$) is to be considered. The main difficulty with using these expressions for the region near the jet inlet is that similarity arguments do not apply here because of the rapidly changing turbulent flow. Therefore, the eddy viscosity formulation expected to apply near the inlet is different than that for far downstream. A number of two-region eddy viscosity expressions, represented by equations (7a) to (7d), are considered here in addition to the constant one-region value from reference 2, given by equations (6a) and (6b).

For the two-region formulations a cutoff rule, such as given by equations (7c) and (7d), must be chosen. The effect of the cutoff rule is shown in the eddy viscosity plots of figure 5. This figure shows three basically different eddy viscosity axial formulations for a typical velocity ratio of $U_e/U_J = 16$. These formulations are included in equations (6a) to (7d). The values of the coefficients are given on the curves in figure 5. The one-region expression is seen to give a much larger eddy viscosity near the inlet than the two-region expressions. The result of using equation (7c) (from ref. 4) rather than equation (7d) as the cutoff rule is a discontinuity of nearly a factor of 6 in the eddy viscosity at about 3 jet radii from the inlet. Fundamentally, neither equation (7c) nor (7d) can be given preference. Equation (7c) follows from a conventional, but arbitrary, definition of the jet potential core region, while equation (7d) is mathematically more satisfying.

Several attempts at choosing the best cutoff rule by comparison with pure air data of reference 5 were inconclusive. The centerline velocity data fit in figure 4(a), with the discontinuous eddy viscosity formulation from reference 4, could be improved only by redefining the potential core cutoff in equation (7c) so that the cutoff value of $(U_e - U_{\mathfrak{t}})/(U_e - U_J)$ was not 0.99 but varied with U_e/U_J. It was felt that such a variable cutoff rule could not be accurately generalized. On the other hand, it was possible to make a cutoff rule comparison with the two-fluid data of reference 5. Calculations with the code using the three eddy viscosity formulations in figure 5 gave significantly different density decay than could be compared with the air-freon ($\rho_e/\rho_J = 0.25$) data. These calculations used the boundary layer inlet velocity profile discussed previously. The

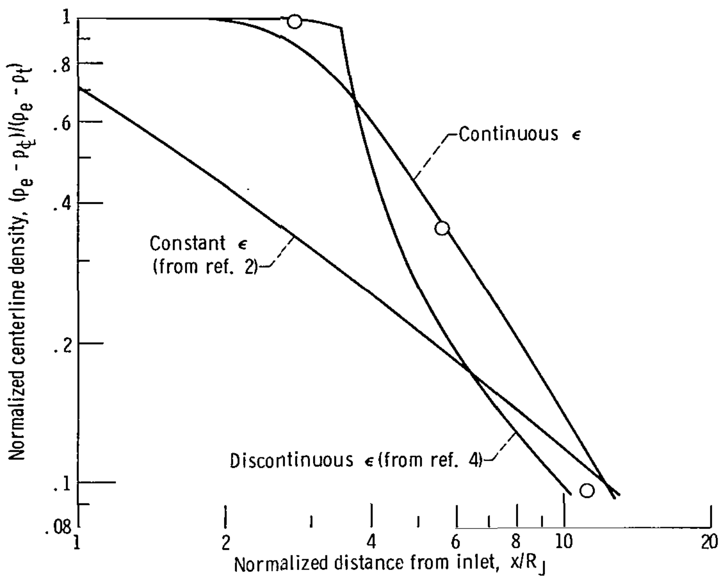


Figure 6. - Centerline density decay using various eddy viscosity formulation from figure 5. Code with boundary layer inlet velocity profile; air-freon data from reference 5; velocity ratio, U_e/U_J = 11.6; density ratio, ρ_e/ρ_J = 0.25.

calculated density profiles and the two-fluid data are shown in figure 6 for the velocity ratio $U_e/U_J = 11.6$. The eddy viscosity coefficients are the same as in figure 5 and were not adjusted for best fit with the two-fluid data.

The shape of the density decay curves shows that the constant eddy viscosity calculation wrongly predicts centerline density decay that starts too close to the inlet. The discontinuous two-region formulation also gives a knee in the density curve, at the point of ϵ discontinuity, that is not found in this data or at other velocity ratios. The continuous two-region formulation gives a density decay most like the data. From these results it is concluded that the continuous two-region formulation for eddy viscosity most nearly represents the data. Equations (7a), (7b), and (7d) were thus used in the final pure air data fits.

Evaluation of the Coefficients in the Axial Eddy Viscosity Formulation

Calculations were made with the jet code using the boundary layer inlet velocity profile and the continuous two-region eddy viscosity formulation given by equations (7a), (7b), and (7d). In these calculations the eddy viscosity coefficients k_0, k_1, and k_2 were varied until a best fit in the centerline velocity decay was found for the entire range of the pure air data.

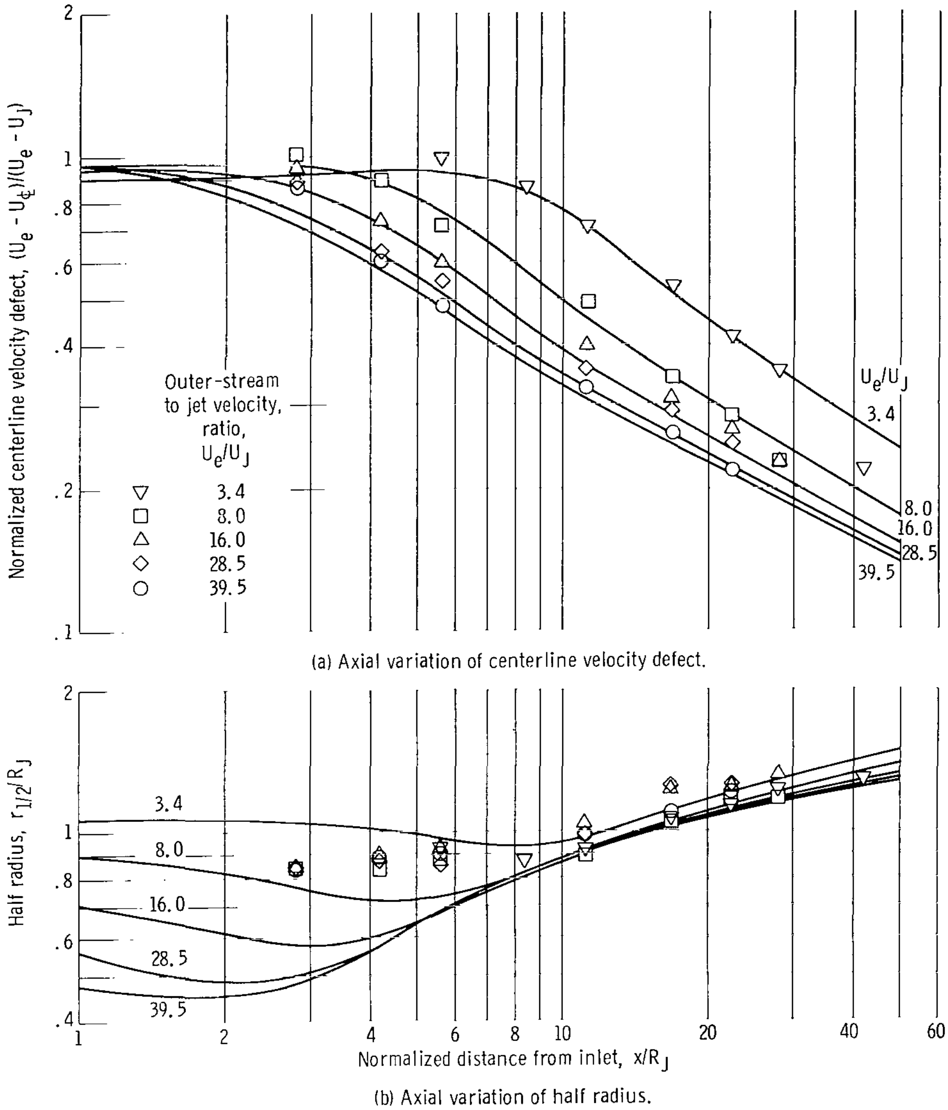


Figure 7. - Velocity defect and half radius variation for various inlet velocity ratios. Code with boundary layer inlet velocity profile and best fit eddy viscosity; data from reference 5.

The values of k_1 that were used ranged from 0.00038 to 0.00300, and those of k_2 ranged from 0.010 to 0.050. These coefficient values included the half-jet value of $k_1 = 0.00137$ and the far-jet value of $k_2 = 0.0256$ given by Schlichting (ref. 8, pp. 599 and 608). Two values of k_0 for turbulence upstream of the inlet were also used. The value $k_0 = 0.014\ U_e/U_J$ was derived for the outer stream boundary layer from the one-seventh power law, and the value $k_0 = 0.005$ was derived for the jet pipe flow from the universal velocity distribution. Calculations with these values of k_0 and those of k_1 and k_2 from reference 8 indicated that for a good data fit the k_0 term in equation (7a) could be ignored. The best fit value of k_2 could be accurately determined independent of k_1 by fitting the far-jet data points beyond about $x/R_J = 10$. This value was found to be $k_2 = 0.0340$, independent of velocity ratio. The best fit value of k_1 was more difficult to determine with the pure air data because of insufficient accurate data points very near the inlet. However, the value $k_1 = 0.00150$ resulted in a reasonably good centerline velocity data fit for all velocity ratios, as is shown in figure 7(a). This value is also confirmed by the two-fluid comparison with the continuous eddy viscosity in figure 6. The final comparisons with the centerline velocity and half radius data using $k_0 = 0$, $k_1 = 0.00150$, and $k_2 = 0.0340$ are shown in figure 7.

A desirable result of this study is that over the very large range of velocity ratios, $U_e/U_J = 3.4$ to 39.5, good agreement between the calculated and measured velocity decay (as shown in fig. 7(a)) was achieved with two simple empirical eddy viscosity coefficients, which were independent of velocity ratio. The agreement in velocity half radius, shown in figure 7(b), using these coefficients is also good except for the high velocity ratio points within about 5.6 jet radii from the inlet. The agreement in the far-jet region is in fact better than expected, considering the large disagreement shown in figure 3(b) for the original code.

Several more complicated eddy viscosity formulations were investigated in order to reduce the remaining discrepancy in the near-jet half radius of figure 7(b). However, no significant improvement in figure 7(b) was obtained. The smaller half-radius values were persistent in these calculations. Of interest is the fact that the discrepancy in half radius occurs at about the same conditions ($U_e/U_J \geq 16$, $x/R_J < 5.6$) where Zawacki and Weinstein (ref. 5) observed inlet recirculation. In addition, Abramovich (ref. 9, p. 173) shows that at sufficiently large velocity ratios the boundary layer assumptions do not apply near the inlet. Thus it is concluded that the near-jet half-radius discrepancy is due to recirculation effects that are not accounted for in this jet boundary layer calculation. The agreement in figure 7(b) would probably be improved with a more complicated calculation, such as a Navier-Stokes equation solution. It is expected that in experiments with a smoother inlet velocity profile, such as a wide gaussian profile, the recirculation could be eliminated at higher velocity ratios. Thus for the smoother inlet velocity profiles the validity of the jet computer codes would extend to higher velocity ratios.

Radial Formulation of Eddy Viscosity

In order to assess the importance of radial variation of eddy viscosity in the calculations, the radial dependence given by equation (8) was used in the computer code. This equation has a radial variation similar to Townsend's wake flow intermittency factor (ref. 10) and accounts for the flow at the edge of the jet being nonturbulent part of the time. Calculations were made with and without the radial decay of eddy viscosity for various velocity ratios. It was found that the shape of the radial velocity profile was completely unaffected, and the dimensionless centerline velocity values were at most only 5 percent lower with the radial eddy viscosity decay. Such a small effect did not justify the complication of a radial eddy viscosity dependence, and it was concluded that a radially constant eddy viscosity was a valid simplification.

Radial Velocity Profile Comparison

The results of the original code (ref. 4) and the final modified code with the boundary layer inlet velocity profile and the best fit eddy viscosity are compared with the data at the typical velocity ratio $U_e/U_J = 16$ in figure 8. This velocity profile plot illustrates the improved data agreement with the final modified code. The better centerline velocity values are mostly due to the new eddy viscosity expression. The better velocity profile widths are mostly the result of accounting for the measured boundary layer inlet profile. The discrepancy between data and the final calculated profile at $x/R_J = 2.8$ is apparently due to the limitations of this jet boundary layer equation solution, as was mentioned in the discussion of figure 7(b).

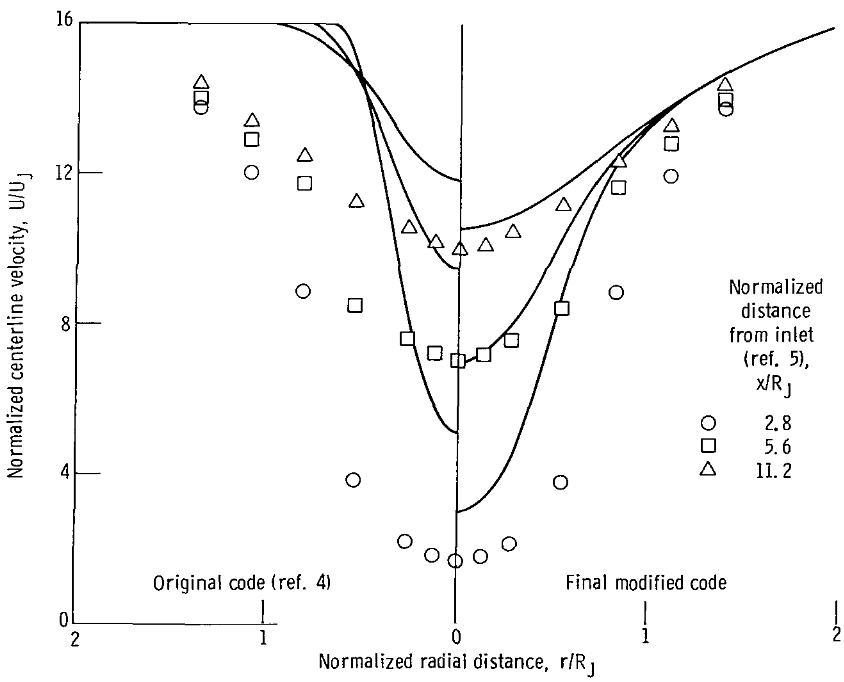


Figure 8. - Calculated velocity profiles compared with air-air data from reference 5. Outer-stream to jet velocity ratio, $U_e/U_J = 16.0$.

SUMMARY OF RESULTS

The coaxial flow computer code of reference 4 was modified and, by comparison with air-air free-jet data from reference 5, a best-fit eddy viscosity formulation was determined. The important results of the present study are the following:

1. The calculated jet half radius (where $(U_e - u) = 1/2(U_e - U_{\mathfrak{L}}))$ was strongly influenced by the specified inlet velocity profile. Therefore, the measured inlet velocity profile was used as input for the code instead of a step profile. For calculations where inlet velocity profile data are not available, it is recommended that a realistic inlet velocity profile be estimated.

2. From calculations with various eddy viscosity formulations, the effects of a radial eddy viscosity dependence were found to be very small. Thus, only the axial dependence appears in the best-fit expression.

3. The best-fit eddy viscosity expression for velocity ratios of $U_e/U_J = 3.4$ to 39.5 was determined to be the following continuous two-region expression:

$$\epsilon_1 = 0.00150\, x(U_e - U_J) \qquad \text{for the near-jet with } x \leq x_{12} \tag{9}$$

$$\epsilon_2 = 0.0340\, r_{1/2}(U_e - U_{\mathfrak{L}}) \qquad \text{for the far-jet with } x > x_{12} \tag{10}$$

The near-jet cutoff x_{12} is the distance from the inlet to where $\epsilon_1 = \epsilon_2$. Note that the half radius $r_{1/2}$ is a variable in equation (10). It must, therefore, be evaluated as part of any calculation that uses these expressions.

4. The largest remaining discrepancy between the final calculations and the data was in the jet half-radius values at $x/R_J < 5.6$ and $U_e/U_J \geq 16$. At most, the calculated half radius is underestimated by 40 percent. This smaller calculated half radius is attributed to the fact the boundary layer solution code does not account for recirculation at the inlet, as was observed experimentally at the high velocity ratios.

5. The results here also apply in general to the two-fluid jet with $\rho_e/\rho_J = 0.25$ to 1.0. It is expected that in a two-fluid data comparison, this same continuous two-region eddy viscosity expression will give good results. However, the coefficients k_1 and k_2 may have to be reevaluated for the two-fluid case to get a more precise data fit.

Lewis Research Center,
National Aeronautics and Space Administration,
Cleveland, Ohio, March 26, 1970,
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